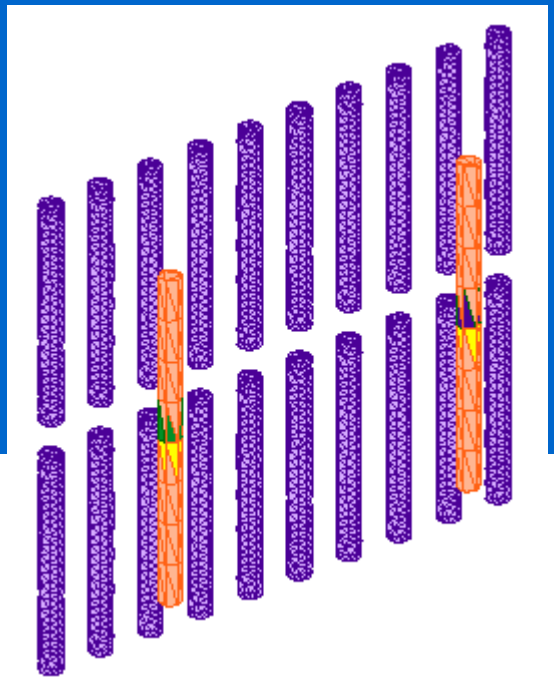
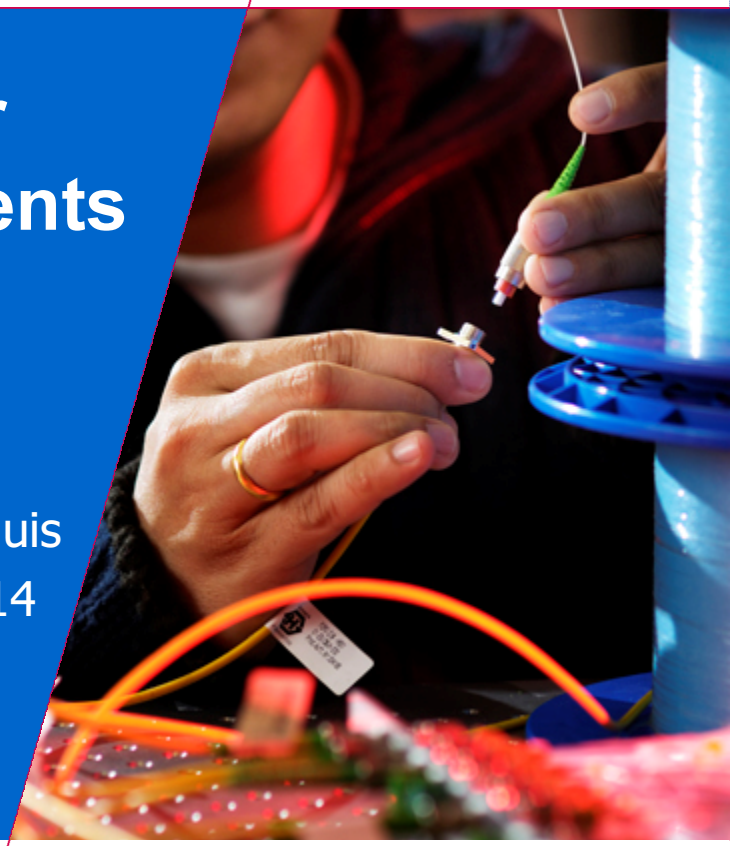
EuCAP 2014

Mixed LEGO-EFIE Approach for the Analysis of Radiating Elements and Anisotropic Media



V. Lancellotti, A. G. Tijhuis

Den Haag, April 10, 2014



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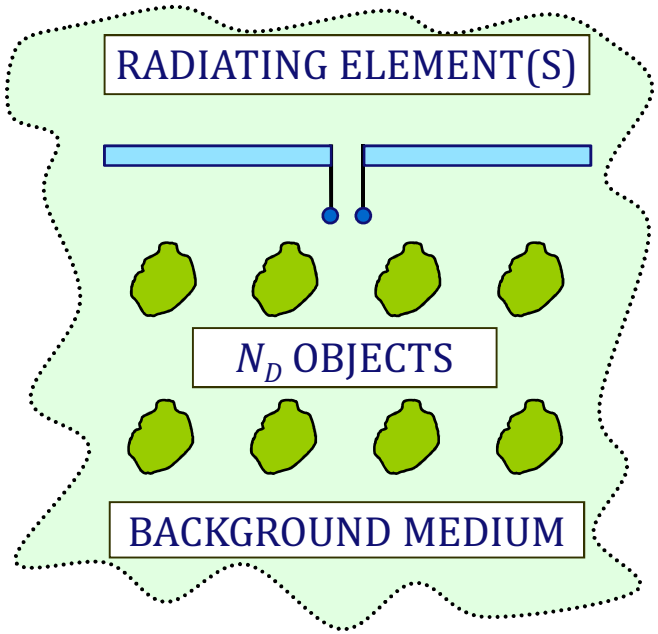
Where innovation starts

Outline

1. The EM problem
2. LEGO modular approach
3. Formulation with LEGO and a modified EFIE
4. Scattering operators with anisotropic bodies
5. Numerical solution: MoM and eigencurrents
6. Validation and convergence
7. Example of Application
8. Conclusion

The electromagnetic problem

1) Anisotropic or inhomogeneous object in a background medium
2) Metallic radiating element(s)
3) Clusters of many anisotropic or inhomogeneous objects



Coupled surface and volume integral equations

MoM

Large matrices

- Linear Embedding via Green's Operators (LEGO) for the objects
- EFIE for the radiating elements

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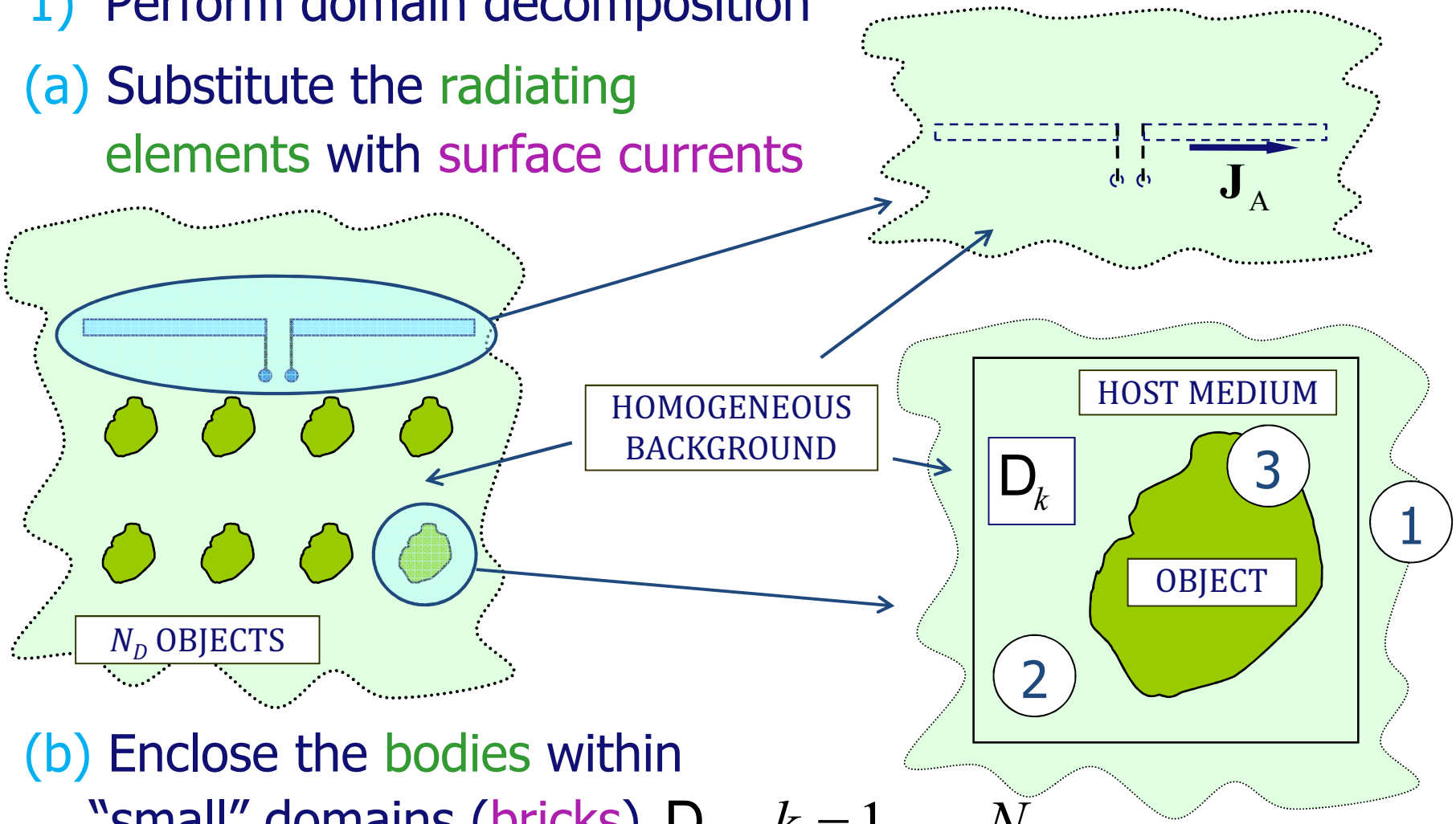
LEGO modular approach (1)

1) Perform domain decomposition

(a) Substitute the radiating elements with surface currents



(b) Enclose the bodies within "small" domains (bricks) $D_k \quad k = 1, \ldots, N_D$

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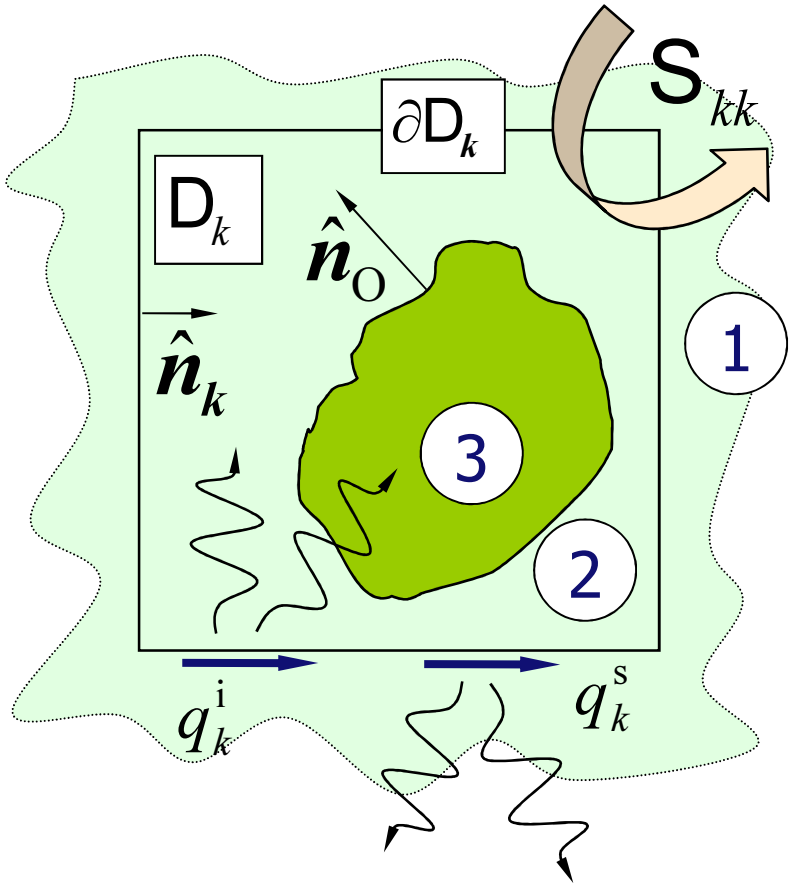
LEGO modular approach (2)

2) Characterize the bricks through scattering operators S_{kk}



$$q_k^{\mathrm{s}} = \mathsf{S}_{kk} q_k^{\mathrm{i}} \qquad k = 1, \ldots, N_D$$

$$q_k^{\mathrm{s,i}} = \begin{bmatrix} \sqrt{\eta_1}\,\mathbf{J}_k^{\mathrm{s,i}} \\ -\mathbf{M}_k^{\mathrm{s,i}} / \sqrt{\eta_1} \end{bmatrix} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

- equivalent currents q_k^{i} yield incident fields inside D_k
- equivalent currents q_k^{s} reproduce scattered fields outside D_k

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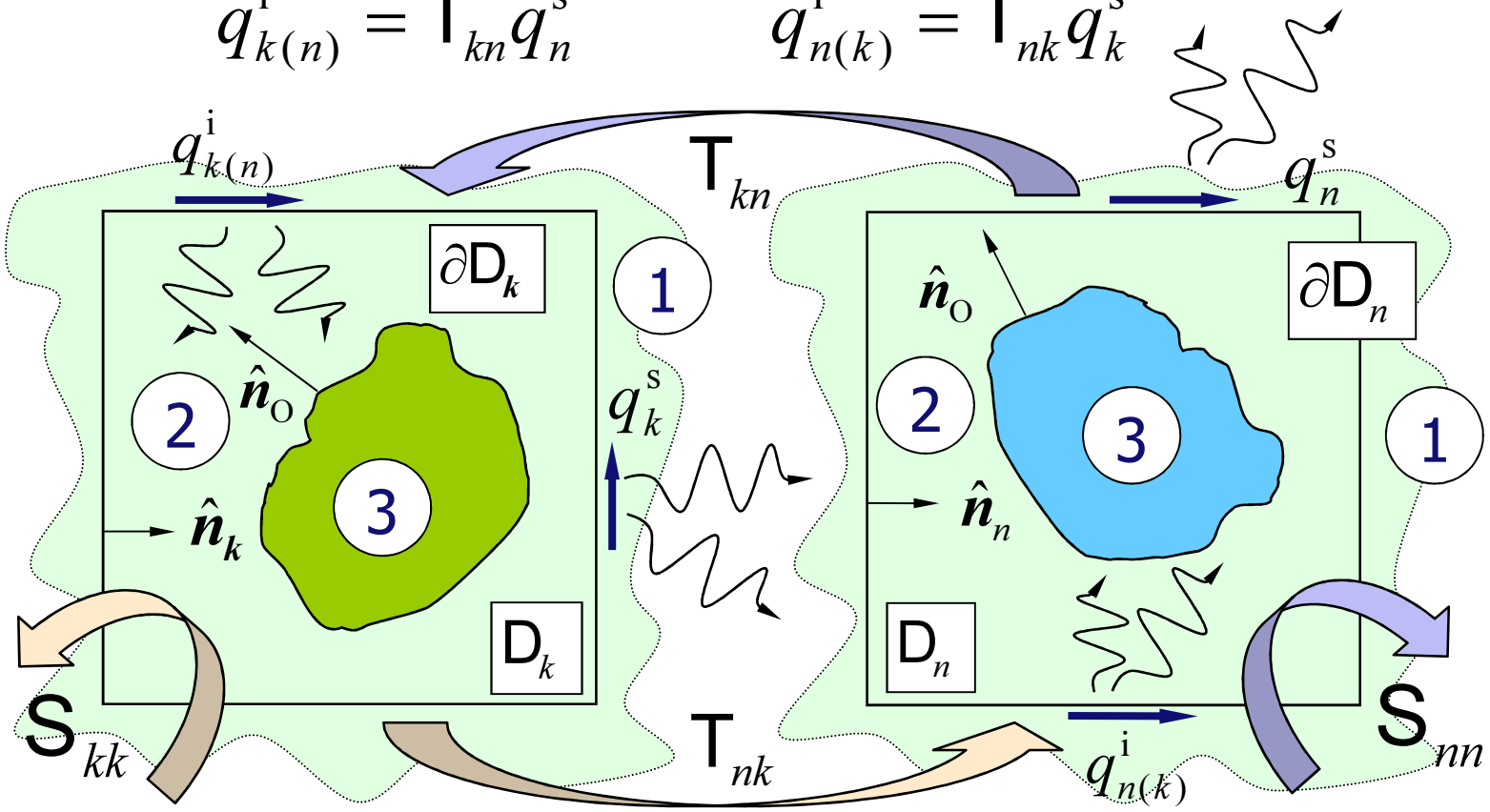
LEGO modular approach (3)

3) Regardless of the brick types, characterize the interactions through transfer operators T_{kn} T_{nk}

$$q^{\mathrm{i}}_{k(n)} = \mathsf{T}_{kn} q^{\mathrm{s}}_{n} \qquad q^{\mathrm{i}}_{n(k)} = \mathsf{T}_{nk} q^{\mathrm{s}}_{k}$$



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LEGO modular approach (4)

4) Combine the bricks electromagnetically

(The bricks interact by contributing additional terms to the incident currents)

$$q_k^{\mathrm{s}} = \mathsf{S}_{kk}\left(q_k^{\mathrm{i}} + \sum_{n \neq k} \mathsf{T}_{kn} q_n^{\mathrm{s}}\right), \quad k,n = 1,\ldots,N_D$$

Total inverse scattering operator S^{-1}

$$\Rightarrow \quad \left(\mathrm{diag}\left\{\mathsf{S}_{nn}^{-1}\right\} - \mathsf{T}\right) q^{\mathrm{s}} = q^{\mathrm{i}}$$

Total transfer operator

$$\mathsf{T} = \begin{bmatrix} 0 & \mathsf{T}_{12} & \cdots & \mathsf{T}_{1N_D} \\ \mathsf{T}_{21} & 0 & \cdots & \mathsf{T}_{2N_D} \\ \vdots & \vdots & \ddots & \vdots \\ \mathsf{T}_{N_D 1} & \mathsf{T}_{N_D 2} & \cdots & 0 \end{bmatrix}$$

Scattered currents

$$q^{\mathrm{s}} = \begin{bmatrix} q_1^{\mathrm{s}} \\ q_2^{\mathrm{s}} \\ \vdots \\ q_{N_D}^{\mathrm{s}} \end{bmatrix}$$

Incident currents

$$q^{\mathrm{i}} = \begin{bmatrix} q_1^{\mathrm{i}} \\ q_2^{\mathrm{i}} \\ \vdots \\ q_{N_D}^{\mathrm{i}} \end{bmatrix}$$

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Formulation (1): radiating elements + objects

a) EFIE over the radiating elements

$$\left[\mathbf{E}^g + \eta_1 \mathsf{L}_\mathrm{A} \mathbf{J}_\mathrm{A} + \sum_{k=1}^{N_D} \mathbf{E}_k^s \right]_{\tan} = 0$$

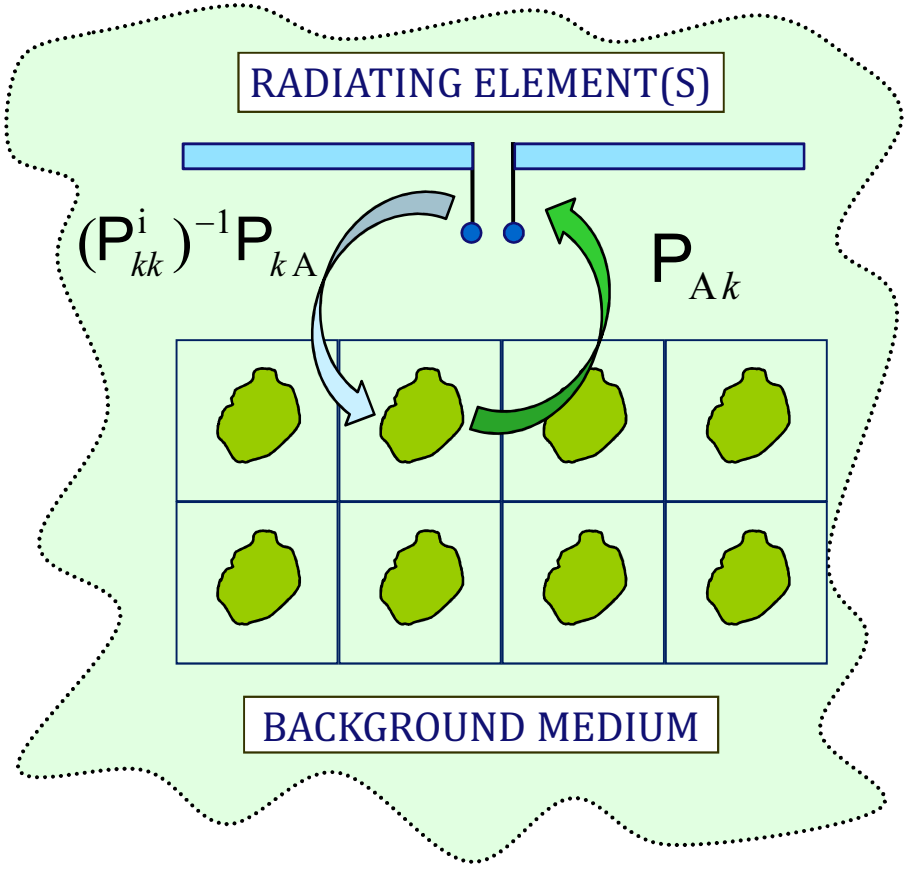
b) LEGO [1] for the objects

$$\mathsf{S}^{-1} q^\mathrm{s} = q^\mathrm{i}$$

c) Coupling relations

$$q_k^\mathrm{i} = \sqrt{\eta_1} (\mathsf{P}_{kk}^\mathrm{i})^{-1} \mathsf{P}_{k\mathrm{A}} \mathbf{J}_\mathrm{A}$$

$$[\mathbf{E}_k^s]_{\tan} = \sqrt{\eta_1} \mathsf{P}_{\mathrm{A}k} q_k^s$$



[1] V. Lancellotti *et al.*, TAP 57 (11) Nov. 2009

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Formulation (2): mixed LEGO-EFIE

d) Eliminate the scattered currents q^s

e) Include the contribution of the objects into the EFIE

CONTRIBUTION OF THE OBJECTS

$$\eta_1(\mathsf{L}_A + \underbrace{\Theta_{AS}\mathsf{S}\mathsf{T}_{SA}})\mathbf{J}_A = -[\mathbf{E}^g]_{\tan}$$

TOTAL SCATTERING OPERATOR OF THE STRUCTURE

PROPAGATION FROM THE ANTENNA TO THE BRICKS

$$\mathsf{T}_{SA} = \begin{bmatrix} (\mathsf{P}^i_{11})^{-1}\mathsf{P}_{1A} \\ \vdots \\ (\mathsf{P}^i_{N_D N_D})^{-1}\mathsf{P}_{N_D A} \end{bmatrix}$$

PROPAGATION FROM THE BRICKS TO THE ANTENNA

$$\Theta_{SA} = [\mathsf{P}_{A1} \quad \cdots \quad \mathsf{P}_{A N_D}]$$

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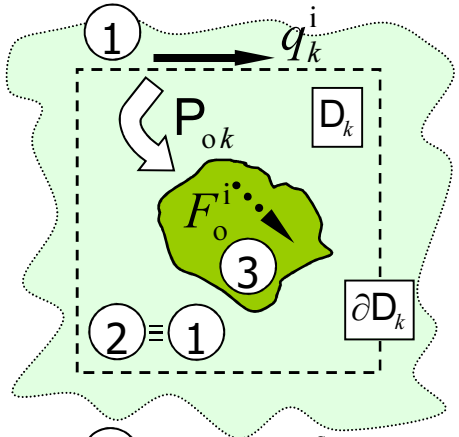
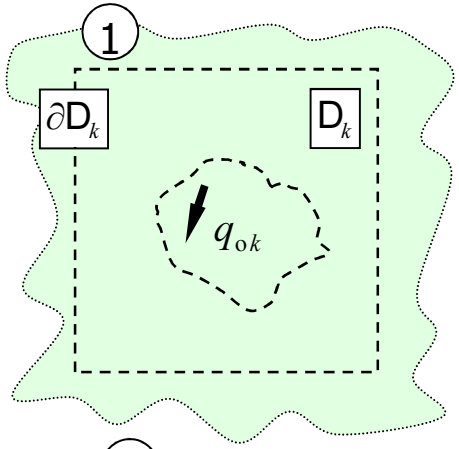
LEGO and anisotropic media

To handle anisotropic bodies [1]

- Modify the way the scattering operators S_{kk} are computed

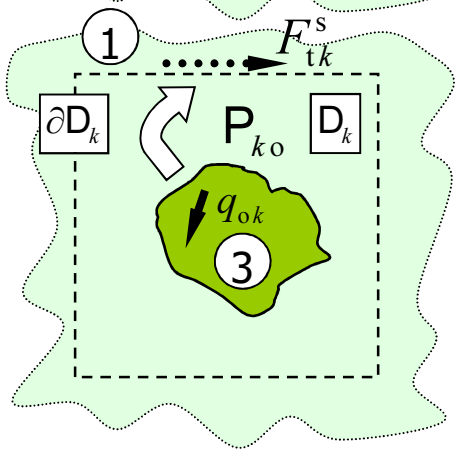


$$F_o^{\mathrm{i}} = \mathsf{P}_{ok} q_k^{\mathrm{i}}$$

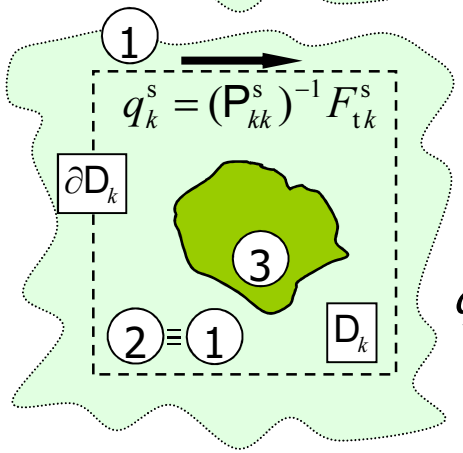


$$\mathsf{X}_{oo} q_{ok} = F_o^{\mathrm{i}}$$

$$q_{ok} = (\mathsf{X}_{oo})^{-1} F_o^{\mathrm{i}}$$



$$F_{tk}^{\mathrm{s}} = \mathsf{P}_{ko} q_o$$



$$q_k^{\mathrm{s}} = (\mathsf{P}_{kk}^{\mathrm{s}})^{-1} F_{tk}^{\mathrm{s}}$$

$$q_k^{\mathrm{s}} = \underbrace{(\mathsf{P}_{kk}^{\mathrm{s}})^{-1} \mathsf{P}_{ko} (\mathsf{X}_{oo})^{-1} \mathsf{P}_{ok}}_{\text{scattering operator}} q_k^{\mathrm{i}}$$

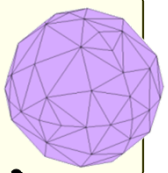
[1] V. Lancellotti *et al.*, IJAP paper # 467931 (2014)

Numerical solution: (1) Method of Moments

- Turn the operators into a weak form through the MoM with sub-domain (SWG/RWG) basis functions

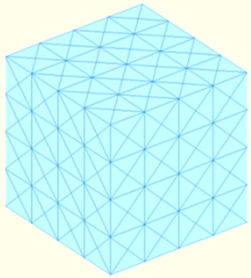
Tetrahedral mesh within an object

$$\mathsf{X}_{oo} \Rightarrow [X_{oo}]$$
$$\mathsf{P}_{ok} \Rightarrow [P_{ok}]$$
$$\mathsf{P}^{\mathrm{s}}_{kk} \Rightarrow [P^{\mathrm{s}}_{kk}] \ldots$$



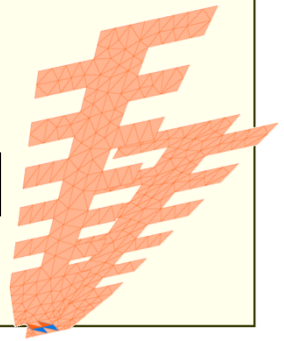
3-D triangular mesh on a brick's boundary

$$\mathsf{S}_{kk} \Rightarrow [S_{kk}]$$
$$\mathsf{T}_{kn} \Rightarrow [T_{nn}]$$
$$q^{\mathrm{s}}_{k} \Rightarrow [q^{\mathrm{s}}_{k}] \ldots$$



3-D triangular mesh on the radiating elements

$$\mathsf{L}_{\mathrm{A}} \Rightarrow [L_{\mathrm{A}}]$$
$$\mathbf{J}_{\mathrm{A}} \Rightarrow [J_{\mathrm{A}}]$$
$$\ldots$$



$$\Rightarrow \eta_1\left([L_{\mathrm{A}}] + [\Theta_{\mathrm{AS}}][S][T_{\mathrm{SA}}]\right)[J_{\mathrm{A}}] = -[E^g]$$

- $[S]^{-1}$ may be too big for direct inversion

  ⇒ use eigencurrents expansion to compute

$$[L_{\mathrm{S}}] = [\Theta_{\mathrm{AS}}][S][T_{\mathrm{SA}}]$$

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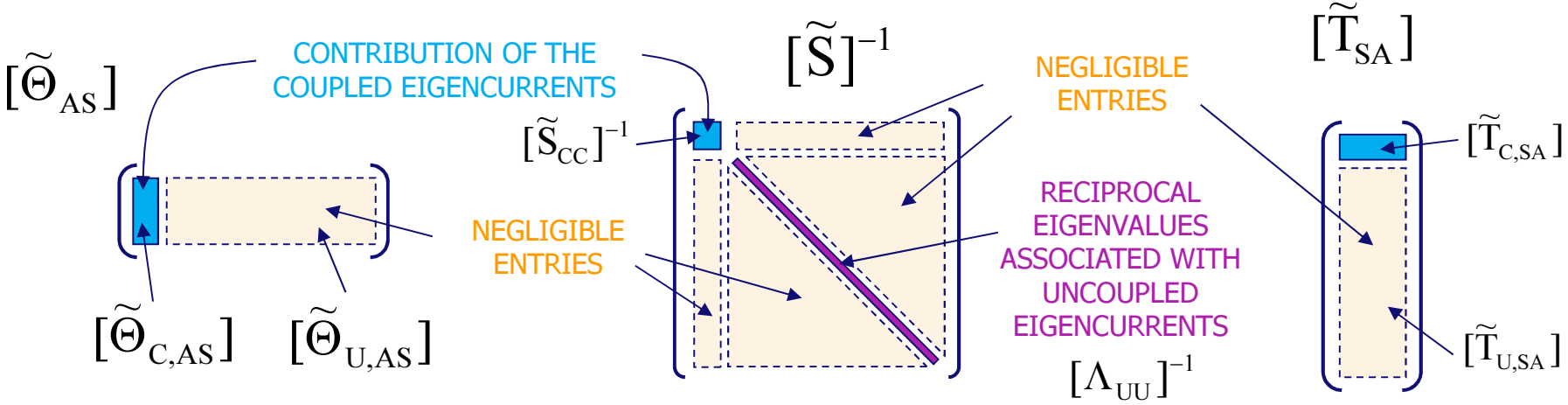
Numerical solution: (2) eigencurrents

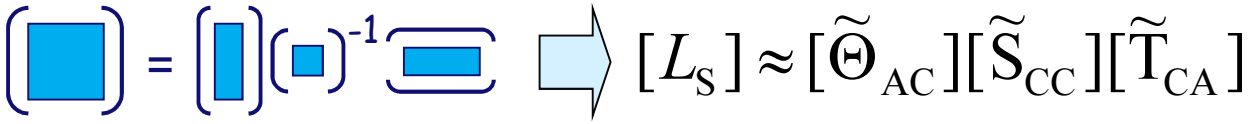
- Use the eigenvectors (eigencurrents) of $[S_{kk}]$

$$[S_{kk}][V_{kk}] = [V_{kk}][\Lambda_{kk}] \quad \Rightarrow \quad [\Psi] = \mathrm{diag}\{[V_{kk}]\}$$

- Separate and rearrange terms corresponding to coupled eigencurrents and uncoupled eigencurrents
- Reduce the size of $[\Theta_{AS}]$ $[S]^{-1}$ $[T_{SA}]$



- Calculation of $[L_S] = [\Theta_{AS}][S][T_{SA}]$



$$[L_S] \approx [\tilde{\Theta}_{AC}][\tilde{S}_{CC}][\tilde{T}_{CA}]$$

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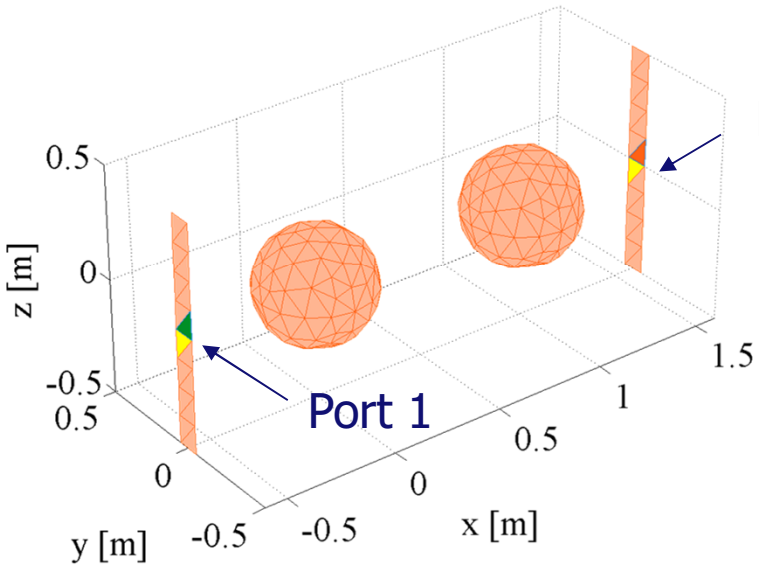
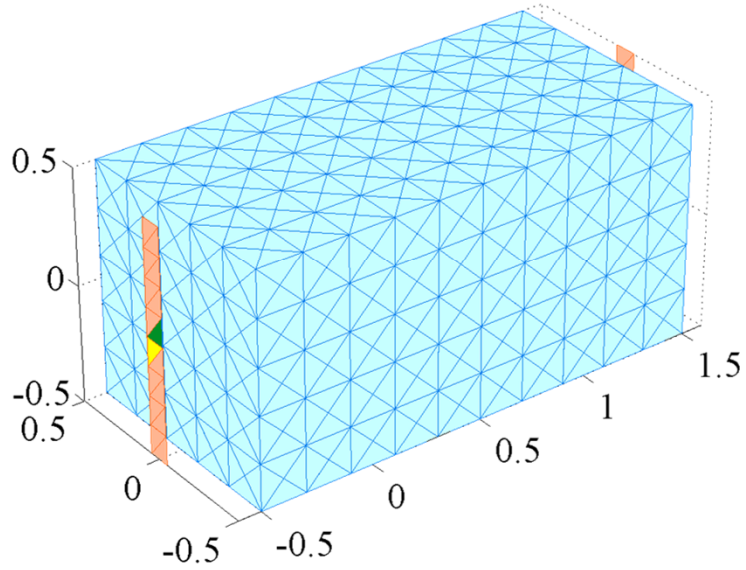
Example of validation (1)

- 2 flat-dipoles and 2 dielectric spheres

Actual EM problem



Model for mixed EFIE-LEGO



- # SWGs in each sphere $N_O = 1834$
- # RWGs on the dipoles $N_A = 30$

- # RWGs on a brick $2N_F = 1800$
- # bricks $N_D = 2$

- # coupled eigencurrents $N_C = 30$

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Example of validation (2)

- 2 flat-dipoles and 2 dielectric spheres

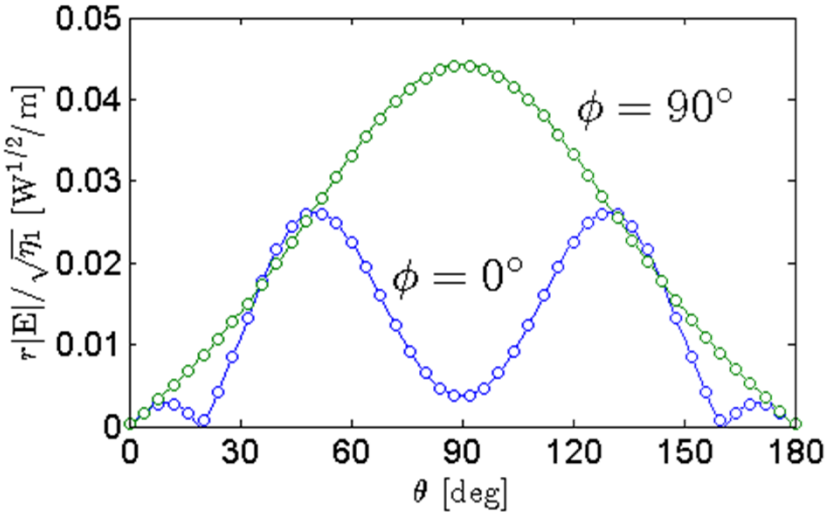
a)(o) present code (SIE/VIE)

b)(-) old LEGO code (PMCHWT)

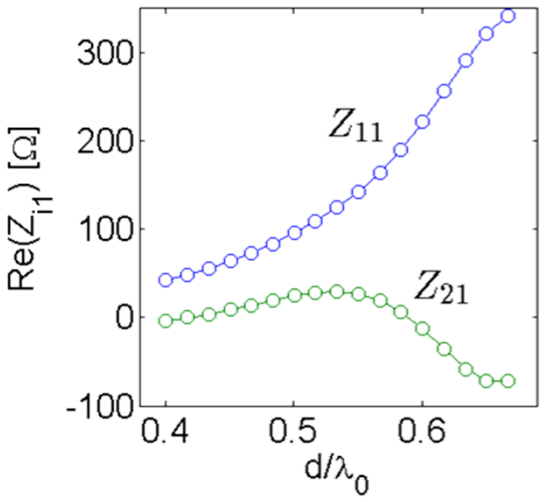
$$\varepsilon_1 = \varepsilon_2 = \varepsilon_0 \qquad \bar{\boldsymbol{\varepsilon}}_3 = 2\varepsilon_0 \bar{\mathbf{I}}$$

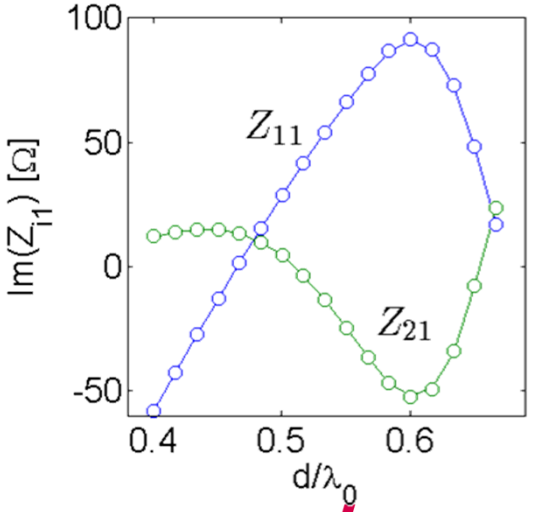
$$V_{G1} = V_{G2} = 1\,\mathrm{V}$$

Radiated field @ $d/\lambda_0 = 0.47$



Impedance matrix





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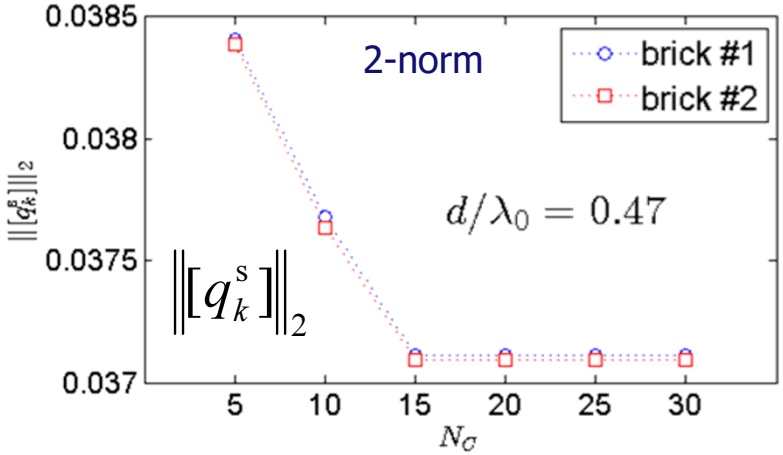
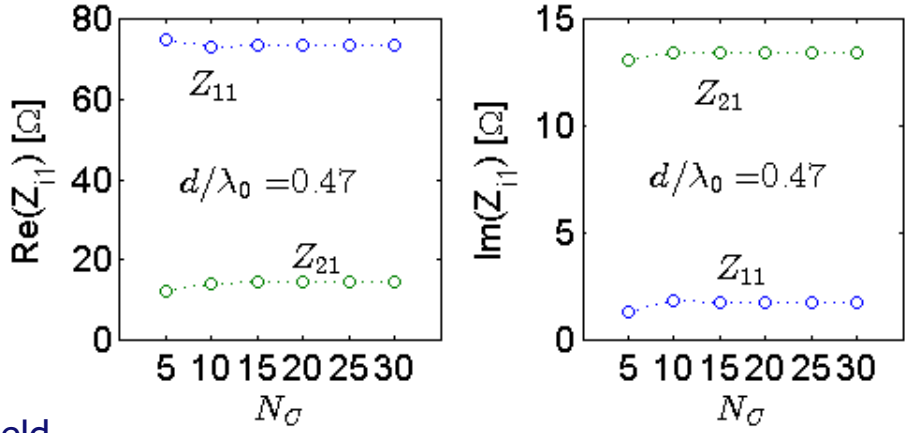
Convergence with # coupled eigencurrents

- 2 flat-dipoles and 2 dielectric spheres $d/\lambda_0 = 0.47$

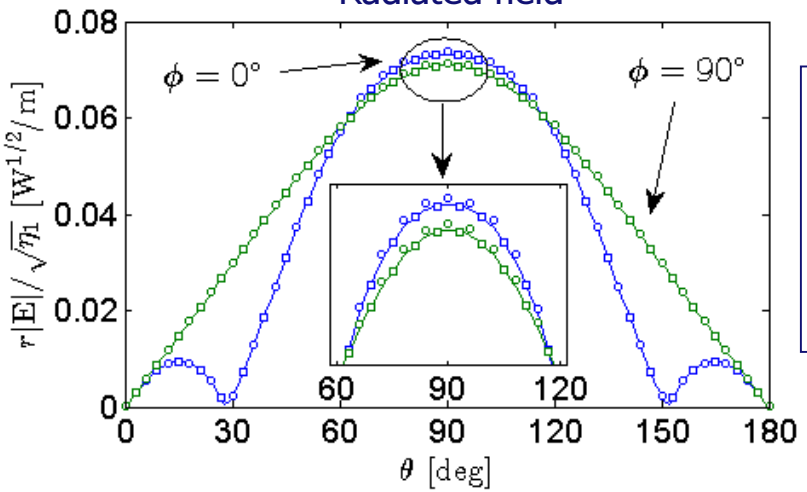
$$\varepsilon_1 = \varepsilon_2 = \varepsilon_0 \quad \bar{\boldsymbol{\varepsilon}}_3 = \varepsilon_0(4\hat{x}\hat{x} + 2\hat{y}\hat{y} + 2\hat{z}\hat{z})$$



Impedance matrix



Radiated field



(o) $N_C = 10$

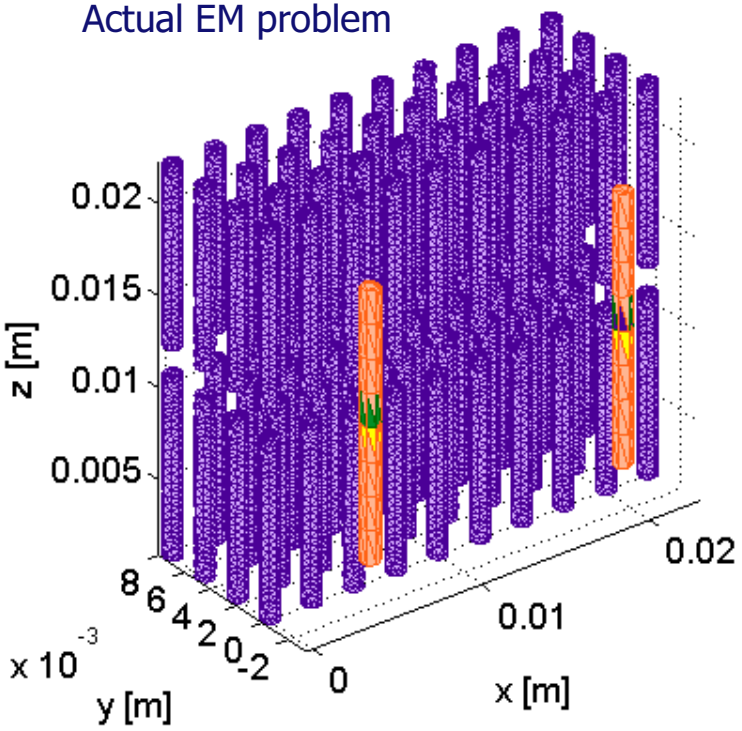
(□) $N_C = 20$

(-) $N_C = 30$

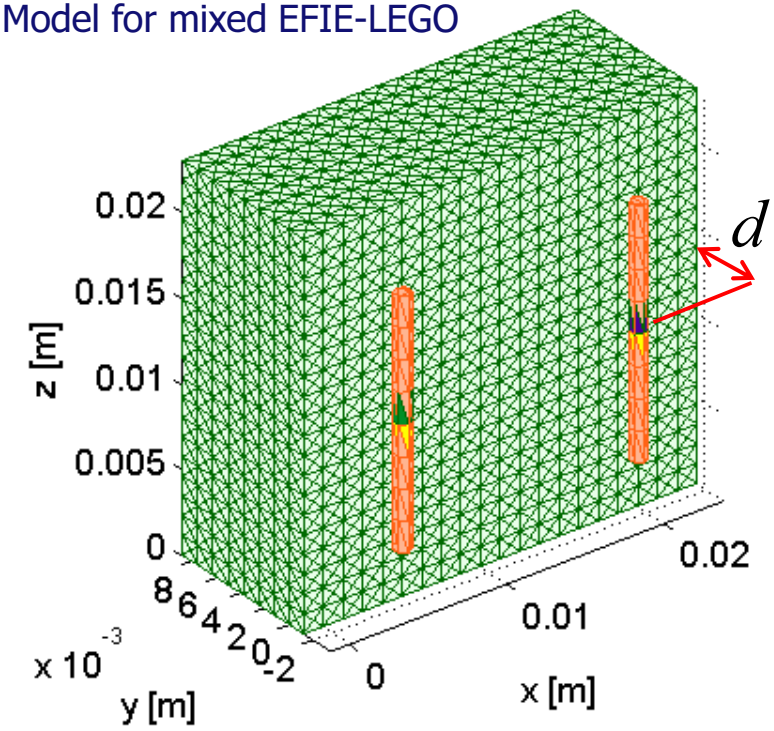
Results: micro plasma rods (1)

- 2 dipole antennas and screens of plasma rods

Actual EM problem



Model for mixed EFIE-LEGO



- # SWGs in each rod $N_O = 1876$
- # RWGs on the dipoles $N_A = 312$

- # RWGs on a brick $2N_F = 1056$
- # bricks $N_D = \{20,40,60,80\}$

- # coupled eigencurrents $N_C = 50$

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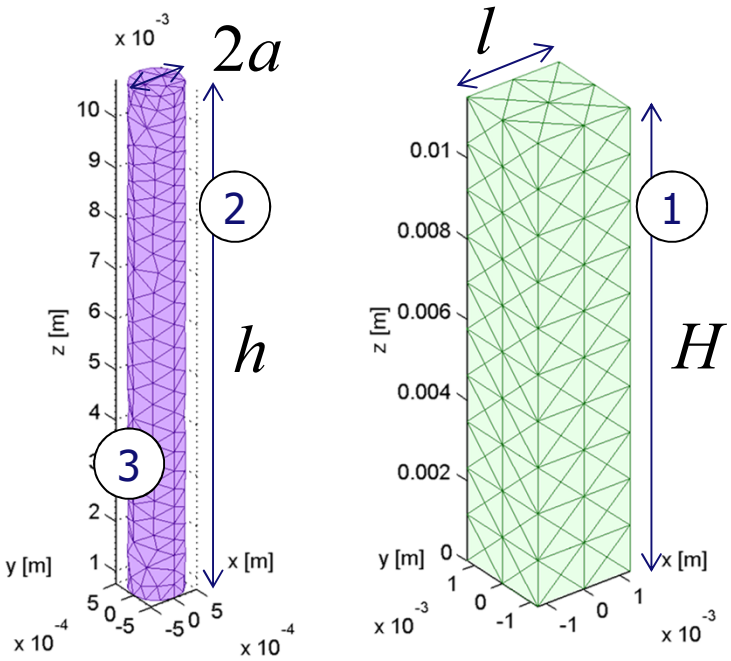
Results: micro plasma rods (2)

- 2 dipole antennas and screen of plasma rods
- magnetized collisional electron plasma



$$n_e = 10^{19}\ \mathrm{m}^{-3} \qquad \boldsymbol{B}_0 = 1\hat{\boldsymbol{z}}\ \mathrm{mT}$$

$$\nu_e = 0.5\omega_{pe} \qquad f \in 10\ \mathrm{GHz}$$

$$2a = 1\ \mathrm{mm} \qquad h = 10\ \mathrm{mm}$$

$$l = 2.5\ \mathrm{mm} \qquad H = 11.5\ \mathrm{mm}$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_0$$

$$\bar{\varepsilon}_3 = \varepsilon_0 \begin{bmatrix} -1.67 - \mathrm{j}3.79 & 7.04\cdot10^{-3} + \mathrm{j}2.5\cdot10^{-3} & 0 \\ -7.04\cdot10^{-3} - \mathrm{j}2.5\cdot10^{-3} & -1.67 - \mathrm{j}3.79 & 0 \\ 0 & 0 & -1.67 - \mathrm{j}3.79 \end{bmatrix}$$

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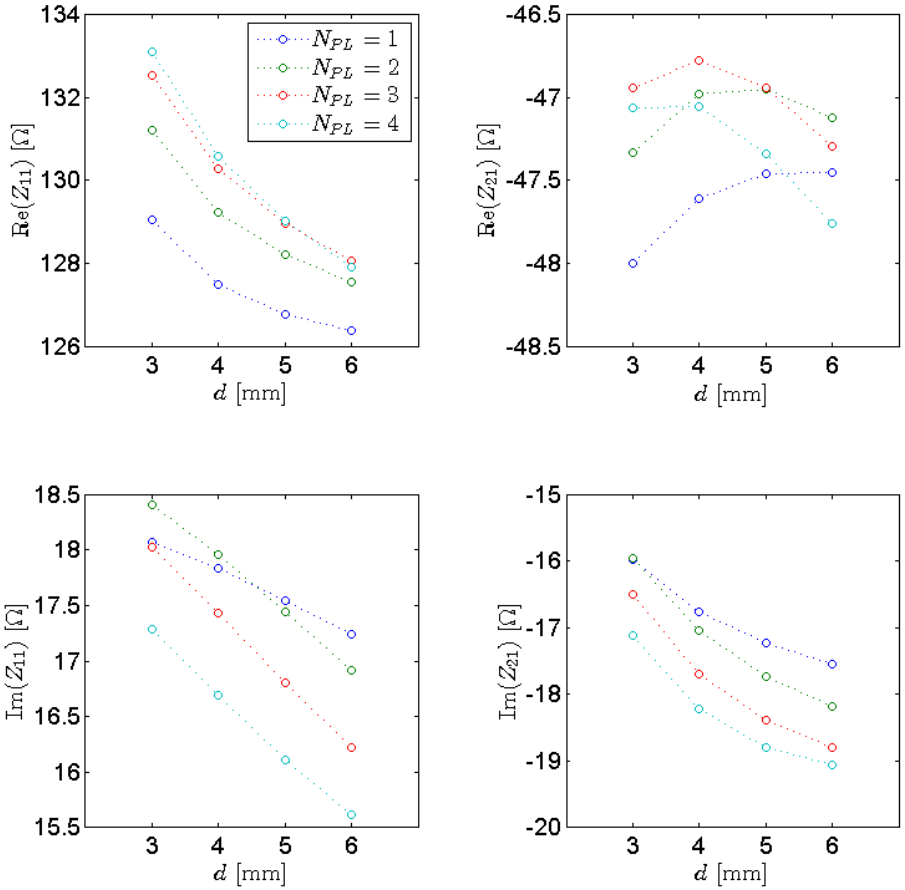
Results: micro plasma rods (3)

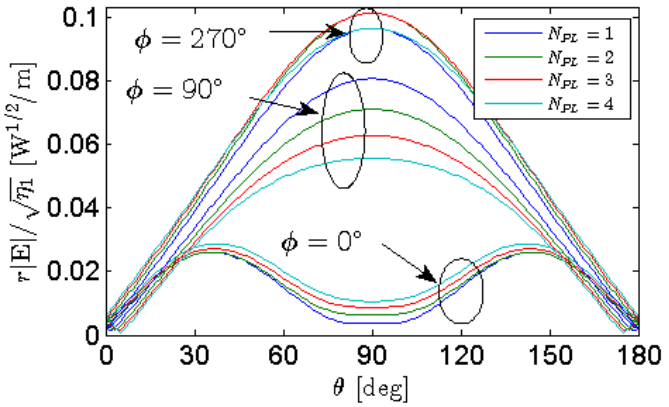
- 2 dipole antennas and screen of plasma rods

$f \in 10\,\text{GHz} \qquad V_{G1} = V_{G2} = 1\,\text{V}$

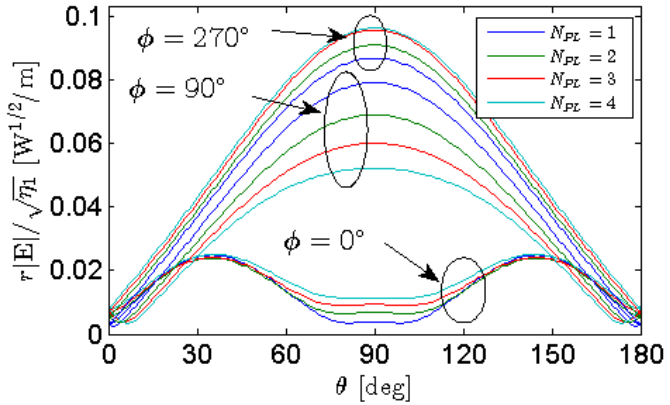
Impedance matrix



Radiated field for $d = 3$ mm



Radiated field for $d = 6$ mm



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Conclusion

- Mixed EFIE-LEGO approach for radiating elements and anisotropic objects
  a) Including anisotropy only affects the calculation of $[S_{kk}]$ (requires volume integral equations) [1]
  b) LEGO bricks can afford a reduction of the number of unknowns *per se* ($2N_F < N_O$)
  c) Eigencurrents expansion provides a mean to compute $[L_S] = [\Theta_{AS}][S][T_{SA}]$ efficiently
- Ongoing work
  - Extension to radiating elements and anisotropic objects with different properties (i.e., shape and constitutive parameters)

[1] V. Lancellotti *et al.*, IJAP paper # 467931 (2014)

Thank you!